

NMC SAMPLE PROBLEMS: GRADE 10

1. Simplify: $(2^{-1} + 3^{-1})^{-1}$
(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{1}{5}$ (d) 5 (e) 6

2. How many factors does 108 have?
(a) 8 (b) 9 (c) 10 (d) 12 (e) None of these

3. If
$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}} = 2017,$$
what is $\frac{a-b}{a+b}$?
(a) $\frac{1}{2017}$ (b) $-\frac{1}{2017}$ (c) 2017 (d) -2017 (e) None of these

4. A palindrome is a number that reads the same backward or forward. For example, 35853 is a palindrome. There is one prime number that is a factor of all palindromes with an even number digits. What is the prime number?
(a) 7 (b) 11 (c) 13 (d) 17 (e) 19

5. The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?
(a) 7 (b) 8 (c) 9 (d) 5 (e) 12

6. Simplify $\sqrt[3]{a^2 \sqrt[4]{a} \sqrt{a}}$, where a is a positive real number.
(a) $a^{19/24}$ (b) $a^{1/2}$ (c) $a^{-1/2}$ (d) $a^{5/2}$ (e) None of these

7. The greatest integer less than $2\sqrt{5} + \sqrt{21}$ is:
(a) 6 (b) 7 (c) 8 (d) 9 (e) 10

8. Suppose α and β are nonzero real solutions of the equation $x^2 - 6x + 2 = 0$. Find the sum $\frac{1}{\alpha} + \frac{1}{\beta}$.
(a) 3 (b) $\sqrt{2}$ (c) 7 (d) $\sqrt{7}$ (e) None of these

9. The binary operation \diamond is defined by

$$a \diamond b = a - \frac{1}{b}.$$

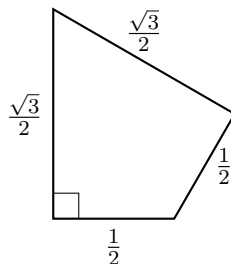
Find the sum of all real numbers x satisfying $(x \diamond x) \diamond x = 1$.

- (a) 2 (b) 1 (c) 0 (d) -1 (e) -2
10. Min climbs a flight of 5 stairs. He can step 1, 2 or 3 stairs at a time. In how many ways can Min climb the stairs?
- (a) 10 (b) 11 (c) 12 (d) 13 (e) 16
11. Compute the perimeter of the triangle whose vertices are $(0,0)$ and two intercepts of the line $5x + 12y = 60$.
- (a) 25 (b) 30 (c) 35 (d) 45 (e) 50
12. Determine $i + i^2 + i^3 + i^4 + \dots + i^{2017}$, where $i^2 = -1$.
- (a) i (b) 1 (c) $-i$ (d) -1 (e) $1 + i$
13. How many consecutive zeros are there at the end of $100!$.
- (a) 18 (b) 100 (c) 25 (d) 24 (e) None of these
14. Find $\sin(\cos^{-1} x)$ in terms of x .
- (a) $\frac{1}{x}$ (b) $\frac{1-x}{x}$ (c) $\sqrt{1-x^2}$ (d) $1-x$ (e) None of these
15. Suppose that $f(3-x) = 2x^2 - 5x + 4$ and $f(x) = ax^2 + bx + c$. What is $a + b + c$?
- (a) -1 (b) 0 (c) 1 (d) 2 (e) 3
16. What is the remainder when 7^{12} is divided by 12?
- (a) 1 (b) 2 (c) 3 (d) 4 (e) 6
17. How many 4-digit numbers are there whose digit sum equals 10? For example, 2017 is such a 4-digit number.
- (a) 200 (b) 210 (c) 219 (d) 220 (e) 286
18. How many real solutions are there to the equation $|2-x| + |x-5| = 3$?
- (a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

19. Suppose that $3 + 2\sqrt{2}$ solves $x^2 - 6x + a = 0$. Find the value of a .
 (a) 1 (b) 3 (c) 5 (d) 6 (e) 8
20. Two numbers, x and y are selected at random from the interval $[0, 3]$. What is the probability that $y \geq x + 1$?
 (a) $2/7$ (b) $1/7$ (c) $1/3$ (d) $4/9$ (e) $2/9$
21. Consider the sequence a_n defined by $a_1 = 1, a_2 = 1 - 2, a_3 = 1 - 2 + 3, \dots, a_n = 1 - 2 + 3 - \dots + (-1)^{n+1}n$. Find

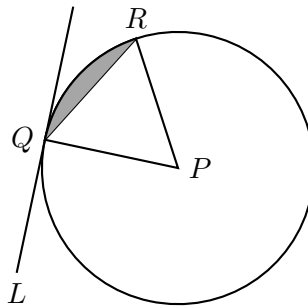
$$\sum_{n=1}^{99} a_n.$$

- (a) 99 (b) 100 (c) 50 (d) 49 (e) 0
22. $\sqrt{7 - \sqrt{40}}$ equals
 (a) $\sqrt{7} - \sqrt{2}\sqrt[4]{10}$ (b) $5\sqrt{10}$ (c) $\sqrt{2} - \sqrt{5}$
 (d) $\sqrt{5} - \sqrt{2}$ (e) None of these
23. If a and b are real numbers, then $(a - b\sqrt{-3})^3$ is real if and only if
 (a) $a = b^2$ or $b = 0$ (b) $a = -b^2$ or $b = 0$ (c) $a = b$ and $b = 0$
 (d) $a = 0$ (e) $a^2 = b^2$ or $b = 0$
24. If $y = 2x + 1$ is tangent to the circle centered at the origin in an xy -plane. Find the radius of the circle.
 (a) 1 (b) $\frac{\sqrt{3}}{3}$ (c) $\frac{\sqrt{5}}{5}$ (d) $\sqrt{2}$ (e) $\frac{\sqrt{2}}{2}$
25. Find the area of the given figure. (*Note: Figure not drawn to scale!*)



- (a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}+1}{4}$ (d) $\frac{\sqrt{3}+1}{2}$ (e) None of these
26. How many integers between 1 and 240 are relatively prime to 240?
 (a) 108 (b) 96 (c) 80 (d) 64 (e) 48

27. Let $a + b = 1$ and $a^3 + b^3 = 4$. Find $a^2 + b^2$.
- (a) -1 (b) 3 (c) $\sqrt{3}$ (d) $\sqrt{3}/2$ (e) None of these
28. The largest integer that divides $n^4 - n^2$ for all n is
- (a) 1 (b) 2 (c) 6 (d) 12 (e) 24
29. $\frac{\sqrt{\frac{6}{5}} + \sqrt{\frac{5}{6}}}{\sqrt{\frac{6}{5}} - \sqrt{\frac{5}{6}}}$ equals:
- (a) 11 (b) $\frac{91}{11}$ (c) 10 (d) 12 (e) $\frac{120}{11}$
30. How many terms are there in the expansion of $(x + y + z + 1)^6$?
- (a) 28 (b) 49 (c) 84 (d) 108 (e) 126
31. If $(x^2 - 5x + 6)^3 - (x - 2)^3$ is factored using only real coefficients, then the number of factors ($x^2 = x \times x$ has two factors) is:
- (a) 2 (b) 3 (c) 4 (d) 5 (e) 7
32. Consider the circle below which has center at P with radius 2. If the acute angle between \overline{QR} and line L is 30 degrees, and the angle between L and \overline{QP} is 90 degrees, find the area of the shaded region. (*Note: Figure not drawn to scale!*)



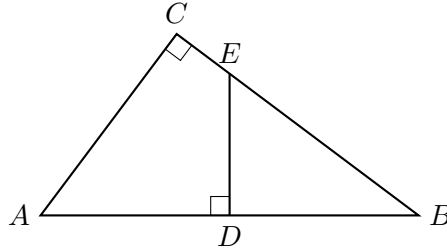
- (a) $2\pi - \sqrt{3}$ (b) $\frac{2\pi - 3\sqrt{3}}{3}$ (c) $\frac{2\pi - 6\sqrt{3}}{3}$ (d) $\frac{4\pi - 3\sqrt{3}}{3}$ (e) None of these
33. Let z be a solution of $x^2 + x + 1 = 0$. What is the value of $z^{10} + z^5 + 1$?
- (a) 0 (b) 3 (c) 1 (d) $i\sqrt{3}$ (e) None of these

34. Let n be the smallest positive integer such that the expansion of $(x^2 + \frac{1}{x^3})^n$ has a constant term. Determine the constant term.
- (a) 10 (b) 12 (c) 15 (d) 20 (e) 21
35. If $\sec \theta - \tan \theta = 2$, what is $\sec \theta + \tan \theta$?
- (a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$ (e) $\frac{1}{3}$
36. How many positive integers d such that the remainder in dividing 4049 by d equals 24?
- (a) 6 (b) 8 (c) 11 (d) 16 (e) None of these
37. Grade A oil sells for \$78 per unit and grade B oil sells for \$54 per unit. If a mixture sells for \$63 per unit, find the ratio of A to B used in the mixture.
- (a) $\frac{11}{15}$ (b) $\frac{2}{3}$ (c) $\frac{3}{5}$ (d) $\frac{3}{4}$ (e) None of these
38. Find the sum of $\frac{1}{10 \cdot 11} + \frac{1}{11 \cdot 12} + \cdots + \frac{1}{19 \cdot 20}$.
- (a) $\frac{1}{10}$ (b) $\frac{1}{11}$ (c) $\frac{1}{19}$ (d) $\frac{1}{20}$ (e) None of these
39. If a ball 2 inches in diameter weighs 32 ounces, then what is the weight of the ball 3 inches in diameter made of the same material?
- (a) 72 (b) 32π (c) 108 (d) 124 (e) None of these
40. If x is a solution to $9^x - 9^{x-1} = 24$, then x satisfies:
- (a) $-1 < x < 0$ (b) $0 < x < 1$ (c) $1 < x < 2$ (d) $2 < x < 3$ (e) None of these
41. Let $1 + i$ be a root of $x^3 + ax^2 + bx - 12 = 0$ where a and b are real numbers. Find $a + b$?
- (a) -5 (b) 5 (c) -6 (d) 6 (e) 0
42. What is the remainder when $4^{10} + 6^{10}$ is divided by 25?
- (a) 12 (b) 8 (c) 4 (d) 2 (e) 1
43. Among the 10 numbered cards, from 1 to 10, we will pick 5 cards randomly. What is the probability that number 7 card is excluded from the selection?
- (a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{5}$ (e) None of these

44. The length of an interior diagonal of a rectangular box is 8 cm, and the sum of the lengths of all the edges of the box is 36 cm. Find the surface area of the box.

(a) 12 cm^2 (b) 14 cm^2 (c) 16 cm^2 (d) 17 cm^2 (e) 19 cm^2

45. In the figure, $AB = 20$, $AC = 12$, $AD = DB$, angles ACB and ADE are right angles. Find the area of the quadrilateral $ADEC$. (Note: Figure not drawn to scale!)



(a) 58.5 (b) 61 (c) 49.5 (d) 62.5 (e) None of these

46. Which one of the following is equal to $\cos \frac{\pi}{5}$?

(a) $\frac{-1+\sqrt{5}}{4}$ (b) $\frac{1+\sqrt{5}}{4}$ (c) $\frac{\sqrt{2}+\sqrt{3}}{4}$ (d) $\frac{-\sqrt{2}+\sqrt{3}}{4}$ (e) None of these

47. The function $g(x) = f(5+x) + f(-1-x)$ is symmetric about the vertical line $x = a$ for all functions f defined on the reals. Find the value a .

(a) -3 (b) -2 (c) 0 (d) 1 (e) 2

48. A sequence begins with a_1, a_2 , and for $n > 2$ is defined by $a_n = a_{n-1} - a_{n-2}$. Find a_{2017} .

(a) $a_1 - a_2$ (b) a_1 (c) a_2 (d) $a_2 - a_1$ (e) $-a_2$

49. Which one of the following is equal to

$$\sqrt[3]{9 - 4\sqrt{5}} + \sqrt[3]{9 + 4\sqrt{5}}?$$

(a) $2\sqrt[3]{3}$ (b) 3 (c) $\sqrt{10}$ (d) $2\sqrt{5}$ (e) 4

50. In a certain village live 27 families. Each family has one, two, or three cars. There are as many families owning three cars as families with only one. How many cars are there in the village?

51. For how many number of real numbers x is $(x + i)^5$ real?

52. Given that the one roots of the equation $x^2 - 2ax + m = 0$ is $a - b$, determine m in terms of a and b . (Completely expand the result.)

53. Find the set of all real numbers x for which

$$2x + \sqrt{4x^2 + 1} - \frac{1}{2x + \sqrt{4x^2 + 1}}$$

is a rational number.

54. Recall that for real number x , $[x]$ denotes the greatest integer not exceeding x . Find all real number pairs (x, y) satisfying the following equations:

$$x + y - [y] = 30.1$$

$$x + [x] + [y] = 71.7$$

55. Compute

$$\sqrt{(19)(20)(21)(22) + 1}.$$

▷ KEYS ◁

[1] (a)	[15] (d)	[29] (a)	[43] (c)
[2] (d)	[16] (a)	[30] (c)	[44] (d)
[3] (b)	[17] (c)	[31] (d)	[45] (a)
[4] (b)	[18] (e)	[32] (b)	[46] (b)
[5] (a)	[19] (a)	[33] (a)	[47] (a)
[6] (a)	[20] (e)	[34] (a)	[48] (b)
[7] (d)	[21] (c)	[35] (d)	[49] (b)
[8] (a)	[22] (d)	[36] (b)	[50] 54
[9] (b)	[23] (e)	[37] (c)	[51] 4
[10] (d)	[24] (c)	[38] (d)	[52] $a^2 - b^2$
[11] (b)	[25] (a)	[39] (c)	[53] The set of all rational numbers.
[12] (a)	[26] (d)	[40] (c)	[54] (29.7, 13.4)
[13] (d)	[27] (b)	[41] (d)	[55] 419
[14] (c)	[28] (d)	[42] (d)	