

NMC SAMPLE PROBLEMS: GRADE 11

1. Which one of the following functions has different domain?
 (a) $\frac{1}{x}$ (b) $\frac{1}{x^{10}}$ (c) $\frac{1}{x^{11}+1}$ (d) $\frac{4}{x\sqrt{x^2+1}}$ (e) $\frac{x-1}{x(x^2+1)}$

2. In the expansion of $(x + y)^8$, what is the coefficient of x^4y^4 ?
 (a) 60 (b) 70 (c) 80 (d) 90 (e) 100

3. Find the equation of the circle centered at $(\frac{1}{2}, 0)$ passing through the point $(0, 1)$.
 (a) $(x - \frac{1}{2})^2 + y^2 = \frac{5}{4}$ (b) $(x + \frac{1}{2})^2 + y^2 = \frac{\sqrt{5}}{2}$
 (c) $x^2 + (y - \frac{1}{2})^2 = \frac{\sqrt{5}}{2}$ (d) $(x - \frac{1}{2})^2 + y^2 = \frac{\sqrt{5}}{2}$
 (e) $x^2 + (y - \frac{1}{2})^2 = \frac{5}{4}$

4. What is the solution set of the following equation: $\ln(x^2 - x) = \ln(2x - 2)$? (Here, $\ln x = \log_e x$)
 (a) $\{0\}$ (b) $\{1\}$ (c) $\{2\}$ (d) $\{1, 2\}$ (e) $\{0, 1\}$

5. Evaluate $\log_{1/2} 8$.
 (a) -3 (b) -2 (c) -1 (d) 1 (e) 2

6. Find the sum of the maximum and minimum values of the expression $-10 \sin(x + 2) + 5$.
 (a) -5 (b) 5 (c) -10 (d) 10 (e) 15

7. Find the sum of all solutions of the equation $|2x - 1| - |x - 5| = 100$.
 (a) 8 (b) -8 (c) 3 (d) -3 (e) 0

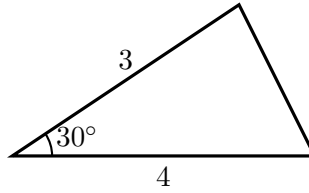
8. Find the polar coordinates (r, θ) for the point with rectangular coordinates $(x, y) = (\sqrt{2}, \sqrt{2})$.
 (a) $(2, \frac{\pi}{4})$ (b) $(\sqrt{2}, \frac{\pi}{4})$ (c) $(2, \pi)$ (d) $(2, \frac{3\pi}{2})$ (e) $(2, 2\pi)$

9. If $10^x = \frac{1}{2}$, then which of the following is 10^{2x-1} ?
 (a) 1 (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{40}$ (e) $\frac{3}{40}$

10. If $\csc x = 10$ and $-90^\circ < x < 90^\circ$, then what is $\cos x$?
 (a) $\frac{3\sqrt{11}}{5}$ (b) $-\frac{3\sqrt{11}}{5}$ (c) $\frac{3\sqrt{11}}{10}$ (d) $-\frac{3\sqrt{11}}{10}$ (e) $\frac{\sqrt{11}}{10}$

11. Find the equation of the straight line which passes through the point $(0, 0)$ and is perpendicular to $83x + 24y + 1 = 0$.
- (a) $83x - 24y = 0$ (b) $24x + 83y = 0$ (c) $24x - 83y = 0$
(d) $24x - 83y + 1 = 0$ (e) $24x + 83y - 1 = 0$
12. Find the vertex point of the parabola $x - y^2 + 1 = 0$.
- (a) $(1, 0)$ (b) $(-1, 0)$ (c) $(0, 1)$ (d) $(0, -1)$ (e) $(-1, -1)$
13. Find the rectangular coordinates (x, y) for the point with polar coordinates $(r, \theta) = (2, \frac{3\pi}{4})$.
- (a) $(-\sqrt{2}, \sqrt{2})$ (b) $(\sqrt{2}, -\sqrt{2})$ (c) $(-\sqrt{2}, -\sqrt{2})$ (d) $(\sqrt{2}, \sqrt{2})$ (e) $(2, 2)$
14. When $\cos \theta = \frac{1}{5}$ and $\tan \theta > 0$, find $\sin \theta$.
- (a) $\frac{4}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{2\sqrt{6}}{5}$ (d) $-\frac{2\sqrt{6}}{5}$ (e) $\frac{3}{5}$
15. Let x and y be real numbers satisfying the following two conditions:
- $$x - y = 1 \quad \text{and} \quad x^3 - y^3 = 16$$
- Find $x^2 + y^2$.
- (a) 9 (b) 10 (c) 11 (d) 12 (e) 13
16. Find the sum $1 + 10 + 10^2 + \dots + 10^{10}$.
- (a) $\frac{10^{10}-1}{9}$ (b) $\frac{10^{10}+1}{9}$ (c) $\frac{10^{10}}{9}$ (d) $\frac{10^{11}+1}{9}$ (e) $\frac{10^{11}-1}{9}$
17. What is the remainder when $1! + 2! + 3! + \dots + 2017!$ is divided by 10?
- (a) 3 (b) 4 (c) 5 (d) 6 (e) 0
18. How many seven digit numbers contain the digit pattern "2017" exactly once?
- (a) 2700 (b) 3000 (c) 3400 (d) 3700 (e) 4000
19. Let α , β and γ be three roots of $x^3 + 1 = 0$. Find the product $p(\alpha)p(\beta)p(\gamma)$, where $p(x) = x^4 + 4x^3 - x + 4$.
- (a) -8 (b) -4 (c) 0 (d) 4 (e) 8

20. Find the area of the following triangle.



- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
21. Suppose a , b , c and d are integers, and $x = \sqrt{1 + \sqrt{2}}$ is a solution of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$. What is the sum $a + b + c + d$?
- (a) 3 (b) -3 (c) 6 (d) -6 (e) -15
22. What is the remainder when $x^{2017} + 2017$ is divided by $x - 1$?
- (a) 2017 (b) 2018 (c) 2019 (d) 2020 (e) 2021
23. Find the value of the product:
- $$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \cdots \left(1 - \frac{1}{100^2}\right)$$
- (a) $\frac{101}{100}$ (b) $\frac{101}{200}$ (c) $\frac{101}{300}$ (d) $\frac{101}{400}$ (e) $\frac{101}{500}$
24. Which one is the number of the real solutions of the equation $e^{-x} - \sin x = 0$?
- (a) 0 (no solutions) (b) 2 (two solutions) (c) 4 (four solutions)
 (d) 6 (six solutions) (e) ∞ (infinitely many solutions)
25. A ball is thrown vertically upward and its height after t seconds is $h(t) = 6.5 + 40t - 32t^2$ feet. Find the maximum height reached by the ball.
- (a) 16 feet (b) 17 feet (c) 18 feet (d) 19 feet (e) 20 feet
26. The function $f(x) = \frac{x}{x-1}$ has one horizontal asymptote, say $y = a$, and one vertical asymptote, say $x = b$. Find $a + b$.
- (a) -1 (b) 0 (c) 1 (d) 2 (e) 3
27. If $\sin x + \cos x = \frac{9}{7}$, then what is the value of $|\sin x - \cos x|$?
- (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{\sqrt{17}}{7}$ (d) $\frac{1}{8}$ (e) $\frac{2}{9}$

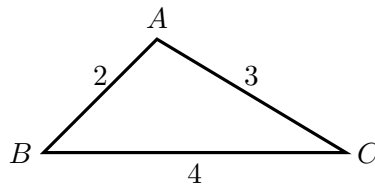
28. Find the remainder when $1 + 2 + 3 + \cdots + 1999 + 2000$ is divided by 1000.
(a) 1008 (b) 1108 (c) 1208 (d) 1308 (e) 0
29. Evaluate $\left(\frac{-1 + \sqrt{3}i}{2}\right)^{100}$
(a) 0 (b) 1 (c) $\frac{-1 + \sqrt{3}i}{2}$ (d) $\frac{-1 - \sqrt{3}i}{2}$ (e) $\frac{1+i}{2}$
30. Let $a_{100}, a_{99}, \dots, a_1$ and a_0 be the coefficients of the expansion of the expression $(x + 1)^{100}$, that is
$$(x + 1)^{100} = a_{100}x^{100} + a_{99}x^{99} + \cdots + a_1x + a_0$$

Find the sum of even coefficients, $a_{100} + a_{98} + \cdots + a_2 + a_0$.
(a) 2^{99} (b) 2^{98} (c) 2^{97} (d) 2^{96} (e) 2^{95}
31. Find the minimum value of $3 \sin x - 4 \cos x + 1$.
(a) 4 (b) 0 (c) -4 (d) -8 (e) -12
32. Find the solution set of the equation $(5^{2x+1})^{x+8} = 5^2$.
(a) $\left\{\frac{-2 \pm \sqrt{29}}{7}\right\}$ (b) $\left\{\frac{-21 \pm \sqrt{29}}{7}\right\}$ (c) $\left\{\frac{19 \pm \sqrt{222}}{7}\right\}$ (d) $\left\{\frac{-17 \pm \sqrt{241}}{4}\right\}$ (e) $\{\}$ (No solution)
33. If $\sin x + \cos x = \frac{1}{2}$, then what is the value of $\sin^3 x + \cos^3 x$?
(a) $-\frac{11}{16}$ (b) $\frac{11}{16}$ (c) $-\frac{3}{8}$ (d) $\frac{3}{8}$ (e) $\frac{1}{8}$
34. There are 2 red ball, 2 blue balls, and 4 white balls in a bag. If James takes out four balls at once, then what is the probability that he takes out at least two white balls?
(a) $\frac{17}{70}$ (b) $\frac{53}{70}$ (c) $\frac{1}{7}$ (d) $\frac{6}{35}$ (e) $\frac{1}{5}$
35. A committee composed of Alice, Mark, Ben, Connie and Francisco is about to select two representatives randomly. What is the probability that Ben is not included in the selection?
(a) $\frac{3}{5}$ (b) $\frac{3}{10}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$ (e) $\frac{2}{5}$
36. Find the sum of all integers between 1 to 1000 (inclusive) that are not multiples of 8.
(a) 500×875 (b) 400×875 (c) 300×875 (d) 200×875 (e) 100×875

37. Evaluate the following ($\ln x = \log_e x$):

$$\ln\left(\frac{1}{2}\right) + \ln\left(\frac{2}{3}\right) + \ln\left(\frac{3}{4}\right) + \cdots + \ln\left(\frac{999}{1000}\right)$$

- (a) $3 \ln 5$ (b) $-3 \ln 5$ (c) $3 \ln 10$ (d) $-3 \ln 10$ (e) 10
38. How many functions which are one-to-one are there from the set $X = \{a, b, c\}$ to the set $Y = \{1, 2, 3, 4\}$?
- (a) 10 (b) 24 (c) 30 (d) 40 (e) 64
39. Let $z = a + i$ be a complex number, where a is a positive real number ($i^2 = -1$). If the real part of z^2 equals that of \bar{z} , then what is a ?
- (a) 1 (b) $\frac{1+\sqrt{5}}{2}$ (c) $\frac{-1+\sqrt{5}}{2}$ (d) $\frac{2+\sqrt{5}}{2}$ (e) 2
40. What is the average of the numbers in the set $\{-7, -2, 3, 8, \dots, 93\}$ given by an arithmetic progression?
- (a) 41 (b) 42 (c) 43 (d) 44 (e) 45
41. The rational function $f(x) = \frac{ax+b}{x}$ has the inverse function $f^{-1}(x) = \frac{3}{2x+1}$. Find $a + b$.
- (a) 1 (b) -1 (c) 0 (d) 2 (e) -2
42. Find $4 \cos B + 3 \cos A$ of the following triangle.



- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
43. An employee of a computer store is paid a base salary \$2,535 a month plus a 5% commission on all sales over \$4,000 during the month (For example, if the gross sales during the month is \$5000, then the commission is \$50). How much must the employee sell in a month to earn a total of \$2,852 for the month?
- (a) \$6,200 (b) \$7,200 (c) \$9,200 (d) \$10,340 (e) \$20,340
44. A speedboat takes a half hour longer to go 8 miles up a river than to return. If the boat cruises at 20 miles per hour in still water, what is the rate of the current? (The units are miles/hour.)
- (a) $4(\sqrt{41} - 4)$ (b) $4\sqrt{2}$ (c) $4(\sqrt{2} - 1)$ (d) 6 (e) 8

45. Which one of the following is the greatest?
(a) 80^5 (b) 63^5 (c) 8^{10} (d) 6^{10} (e) 3^{20}
46. Suppose that a, b are integers and $1 - \sqrt{5}$ is a solution of the equation $x^3 + x^2 + ax + b = 0$. Find $a + b$.
(a) 10 (b) -10 (c) 22 (d) -22 (e) 0
47. Harry performed the calculation: $5 \times 8 = 44$. It turns out Harry's calculation is correct in base b . Find the value b .
(a) 9 (b) 10 (c) 11 (d) 12 (e) 13
48. Suppose that $f(x)$ is a polynomial with integer coefficients, having 100 and 200 as zeros. Which of the following could possibly be the value of $f(150)$?
(a) 2 (b) $2^2 \cdot 5$ (c) 5^2 (d) $2 \cdot 5^4$ (e) $2^3 \cdot 5^4$
49. A polynomial $f(x) = x^3 + ax^2 + bx + c$ satisfies $f(1) = 1$, $f(2) = 2$ and $f(3) = 3$. What is the remainder when $f(x)$ is divided by $x - 4$?
(a) 10 (b) 5 (c) 0 (d) -5 (e) -10
50. Evaluate $\frac{2017^4 + 3 \cdot 2017^3 - 3 \cdot 2017^2 + 6 \cdot 2017 + 8}{2017^3 - 2017^2 + 2017 + 2}$.
(a) 2017 (b) 2018 (c) 2019 (d) 2020 (e) 2021
51. The domain of the function
$$f(x) = \log_{10}(\log_2(\log_3(\log_5 x)))$$
is $\{x|x > c\}$. What is the value of c ?
(a) 0 (b) 1 (c) 10 (d) 100 (e) 125
52. Find the remainder when 1234^{4321} is divided by 5.
53. If $\log_9 4 = a$, then what is $\log_{16} 81$ in terms of a ?
54. Let a and b be integers which satisfy $\sqrt{43 + 30\sqrt{2}} = a + b\sqrt{2}$. Find $a + b$.

55. Let f be a function satisfying the following equation:

$$f(x) = \pi x f(1 - x) \text{ for all real numbers } x.$$

Find $f(100)$.

56. How many ordered pairs (x, y) of real numbers are there which satisfy the following equations?

$$x^{100} + y^{100} = x^{101} + y^{101} = x^{102} + y^{102}.$$

57. The sides of a right triangle form an arithmetic progression. Find the ratio of the shorter leg to the hypotenuse.

▷ KEYS ◁

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|----------|----------|----------|--------------------|
| [1] (c) | [16] (e) | [31] (c) | [46] (d) |
| [2] (b) | [17] (a) | [32] (d) | [47] (a) |
| [3] (a) | [18] (d) | [33] (b) | [48] (e) |
| [4] (c) | [19] (e) | [34] (b) | [49] (a) |
| [5] (a) | [20] (c) | [35] (a) | [50] (e) |
| [6] (d) | [21] (b) | [36] (a) | [51] (e) |
| [7] (b) | [22] (b) | [37] (d) | [52] 4 |
| [8] (a) | [23] (b) | [38] (b) | [53] $\frac{1}{a}$ |
| [9] (d) | [24] (e) | [39] (b) | [54] 8 |
| [10] (c) | [25] (d) | [40] (c) | [55] 0 |
| [11] (c) | [26] (d) | [41] (a) | [56] 4 |
| [12] (b) | [27] (c) | [42] (b) | [57] $\frac{3}{5}$ |
| [13] (a) | [28] (e) | [43] (d) | |
| [14] (c) | [29] (c) | [44] (a) | |
| [15] (c) | [30] (a) | [45] (e) | |